

ESCOAMENTO COMPRESSÍVEL

EXERCÍCIO

PROPRIEDADES DE ESTAGNAÇÃO APÓS CHOQUE NORMAL

Um míssil rombudo voa a Mach 2 ao nível do mar. Calcule a temperatura e pressão no nariz do míssil

$$M_2^2 = \frac{1 + \left[\frac{k-1}{2}\right] M_1^2}{k M_1^2 - \frac{k-1}{2}}$$

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{(k+1)M_1^2}{2 + (k-1)M_1^2}$$

$$\frac{P_2}{P_1} = 1 + \frac{2k}{k+1}(M_1^2 - 1)$$

$$\frac{T_2}{T_1} = \frac{h_2}{h_1} = \left[1 + \frac{2k}{k+1}(M_1^2 - 1) \right] \left[\frac{2 + (k-1)M_1^2}{(k+1)M_1^2} \right]$$

$$s_2 - s_1 = -R \ln \frac{P_{o2}}{P_{o1}}$$

Tabela 1.2 – Tabela para Choque Normal. $k = 1,4$

M_1	M_2	$\frac{\rho_2}{\rho_1}$	$\frac{P_2}{P_1}$	$\frac{T_2}{T_1}$	ds	$\frac{P_{o2}}{P_{o1}}$	$\frac{P_{o2}}{P_1}$
1,00	1,000000	1,000000	1,000000	1,000000	0,000000	1,000000	1,892929
1,02	0,980519	1,033441	1,047133	1,013249	0,000003	0,999990	1,937897
1,04	0,962025	1,067088	1,095200	1,026345	0,000022	0,999923	1,984423
1,06	0,944445	1,100921	1,144200	1,039312	0,000072	0,999751	2,032453
1,08	0,927713	1,134925	1,194133	1,052170	0,000163	0,999431	2,081942
1,10	0,911770	1,169082	1,245000	1,064938	0,000308	0,998928	2,132847
1,12	0,896563	1,203377	1,296800	1,077634	0,000513	0,998213	2,185134
1,14	0,882042	1,237793	1,349533	1,090274	0,000787	0,997261	2,238769
1,16	0,868162	1,272315	1,403200	1,102872	0,001135	0,996052	2,293725
1,18	0,854884	1,306927	1,457800	1,115441	0,001563	0,994569	2,349977
1,20	0,842170	1,341615	1,513333	1,127994	0,002074	0,992798	2,407502
1,22	0,829986	1,376364	1,569800	1,140541	0,002673	0,990731	2,466280
1,24	0,818301	1,411160	1,627200	1,153094	0,003361	0,988359	2,526294
1,26	0,807085	1,445989	1,685533	1,165661	0,004140	0,985677	2,587527
1,28	0,796312	1,480839	1,744800	1,178251	0,005014	0,982682	2,649964
1,30	0,785957	1,515695	1,805000	1,190873	0,005982	0,979374	2,713594

Formulação

Gráficos

Calculadora Online

$$M_2^2 = \frac{1 + [\frac{k-1}{2}]M_1^2}{kM_1^2 - \frac{k-1}{2}}$$

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{(k+1)M_1^2}{2 + (k-1)M_1^2}$$

$$\frac{P_2}{P_1} = 1 + \frac{2k}{k+1}(M_1^2 - 1)$$

$$\frac{T_2}{T_1} = \frac{h_2}{h_1} = \left[1 + \frac{2k}{k+1}(M_1^2 - 1) \right] \left[\frac{2 + (k-1)M_1^2}{(k+1)M_1^2} \right]$$

$$s_2 - s_1 = -R \ln \frac{P_{o2}}{P_{o1}}$$

taboas

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1,00	1,000000	1,000000	1,000000	1,000000	0,000000	1,000000	1,892929
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Hipóteses do modelo

Corpo rombudo $\rightarrow$ Choque em arco

Regime permanente

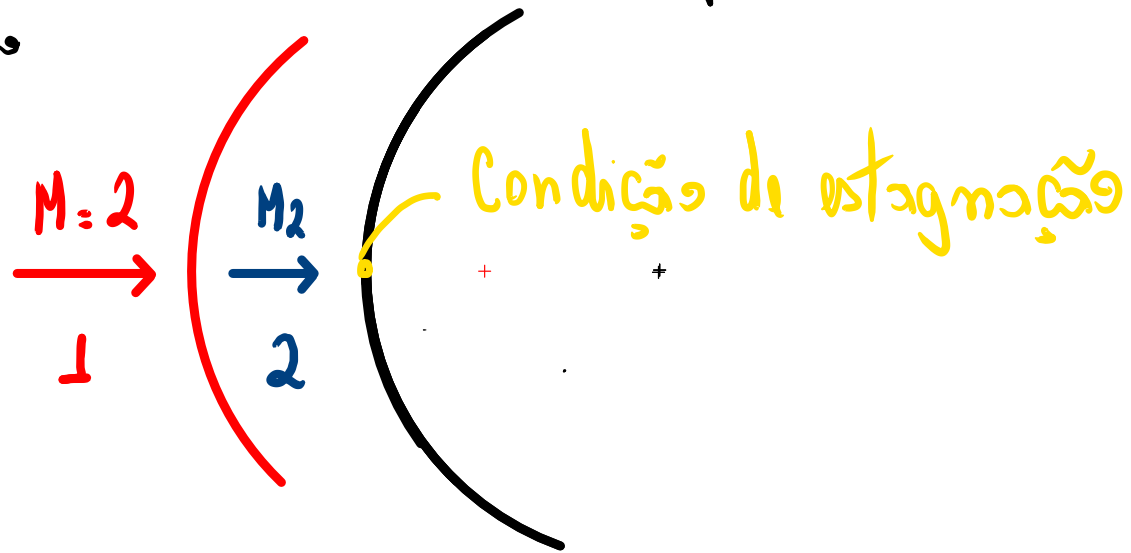
Escoamento unidimensional

Desprezando forças DE corpo

Propriedades contínuas antes e depois do choque

Adiabático

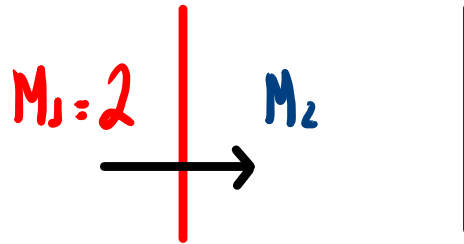
} Choque normal



Um míssil rombudo voa a Mach 2 ao nível do mar. Calcule a temperatura e pressão no nariz do míssil

CONSIDERAR $T = 15^\circ\text{C} = 288,16\text{K}$

$$P = 101,325\text{ kPa}$$



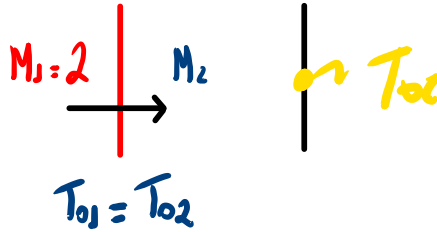
$$h_{01} = h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} = h_{02}$$

$$c_p \cdot T_{01} = c_p T_{02}$$

$$T_{01} = T_{02}$$

Um míssil rombudo voa a Mach 2 ao nível do mar. Calcule a temperatura e pressão no nariz do míssil

Considerando $T = 15^\circ\text{C} = 288,16\text{K}$ $P = 101,325\text{ kPa}$



$$\frac{T_{01}}{T_1} = 1 + \frac{k-1}{2} M_1^2$$

$$h_0 = h + \frac{V^2}{2}$$

$$\frac{T_{01}}{T_1} = 1 + \frac{1,4-1}{2} \cdot 2^2 = 1,8$$

$$T_{01} = 518,688\text{K} = T_{02}$$

Calculando pelas propriedades após o choque: M_2

$$M_2^2 = \frac{1 + \left[\frac{k-1}{2}\right]M_1^2}{kM_1^2 - \frac{k-1}{2}}$$

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{(k+1)M_1^2}{2 + (k-1)M_1^2}$$

$$\frac{P_2}{P_1} = 1 + \frac{2k}{k+1}(M_1^2 - 1)$$

$$\frac{T_2}{T_1} = \frac{h_2}{h_1} = \left[1 + \frac{2k}{k+1}(M_1^2 - 1)\right] \left[\frac{2 + (k-1)M_1^2}{(k+1)M_1^2}\right]$$

$$s_2 - s_1 = -R \ln \frac{P_{o2}}{P_{o1}}$$

$$\begin{matrix} T_1 \\ \rho_1 \end{matrix} \quad M_1 = 2 \quad \xrightarrow{\quad} \quad \begin{matrix} T_2 \\ \rho_2 \end{matrix} \quad \approx \quad \begin{matrix} T_{o2} \\ \rho_{o2} \end{matrix}$$

$$M_2^2 = \frac{1 + \left(\frac{1.4-1}{2}\right) \cdot 2^2}{1.4 \cdot 2^2 - \frac{1.4-1}{2}}$$

$$M_2 = 0.57135$$

Calculo de P2

$$M_2^2 = \frac{1 + [\frac{k-1}{2}]M_1^2}{kM_1^2 - \frac{k-1}{2}}$$

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{(k+1)M_1^2}{2 + (k-1)M_1^2}$$

$$\frac{P_2}{P_1} = 1 + \frac{2k}{k+1}(M_1^2 - 1)$$

$$\frac{T_2}{T_1} = \frac{h_2}{h_1} = \left[1 + \frac{2k}{k+1}(M_1^2 - 1) \right] \left[\frac{2 + (k-1)M_1^2}{(k+1)M_1^2} \right]$$

$$s_2 - s_1 = -R \ln \frac{P_{o2}}{P_{o1}}$$

$$\frac{P_2}{P_1} = 1 + \frac{2 \times 1,4}{1,4 + 1} (2^2 - 1)$$

$$\frac{P_2}{P_1} = 4,5 \approx P_2 = 455,9625 \text{ kPa}$$

Calculo de T2

$$M_2^2 = \frac{1 + \left[\frac{k-1}{2}\right] M_1^2}{k M_1^2 - \frac{k-1}{2}}$$

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{(k+1)M_1^2}{2 + (k-1)M_1^2}$$

$$\frac{P_2}{P_1} = 1 + \frac{2k}{k+1}(M_1^2 - 1)$$

$$\frac{T_2}{T_1} = \frac{h_2}{h_1} = \left[1 + \frac{2k}{k+1}(M_1^2 - 1) \right] \left[\frac{2 + (k-1)M_1^2}{(k+1)M_1^2} \right]$$

$$s_2 - s_1 = -R \ln \frac{P_{o2}}{P_{o1}}$$

$$\frac{T_2}{T_1} = 4,5 \left(\frac{2 + (1,4-1) 2^2}{(1,4+1) \cdot 2^2} \right)$$

$$\frac{T_2}{T_1} = 1,6815 \approx T_2 = 486,27 \text{ K}$$

Propriedades de Estagnação após o choque



$$\left. \begin{array}{c} T_1 \\ P_1 \end{array} \right| \xrightarrow{M_1=2} \left. \begin{array}{c} T_2 \\ P_2 \end{array} \right| \approx \left. \begin{array}{c} T_{02} \\ P_{02} \end{array} \right|$$

$$M_1 = 2 \approx M_2 = 0,57735$$

$$\frac{T_{02}}{T_1} = 1 + \frac{k-1}{2} M_1^2 = 1 + \frac{1,4-1}{2} \cdot 0,57735^2$$

$$\frac{T_{02}}{T_1} = 1,0666 \approx T_{02} = 516,688 \text{ K} = T_{01}!$$

$$\frac{P_{02}}{P_2} = \left(\frac{T_{02}}{T_2} \right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_{02}}{P_2} = 12534 \sim P_{02} = 571,5175 \text{ MPa}$$

TABELAS

Tabela 1.2 – Tabela para Choque Normal. $k = 1,4$

M_1	M_2	$\frac{\rho_2}{\rho_1}$	$\frac{P_2}{P_1}$	$\frac{T_2}{T_1}$	ds	$\frac{P_{o2}}{P_{o1}}$	$\frac{P_{o2}}{P_1}$
2,00	0,577350	2,666667	4,500000	1,687500	0,093933	0,720874	5,640441
2,02	0,573972	2,696181	4,593800	1,703817	0,097678	0,711527	5,743338
2,04	0,570679	2,725463	4,688533	1,720271	0,101473	0,702180	5,847268
2,06	0,567467	2,754511	4,784200	1,736860	0,105317	0,692839	5,952256

$$M_1 = 2 \leadsto M_2 = 0,57735$$

$$\frac{P_{o2}}{P_1} = 5,640441 \leadsto P_{o2} = 571,5177 \text{ kPa} \quad (571,5175)$$

$$\frac{T_2}{T_1} = 1,6875 \quad (1,6815) \leadsto T_2 = 486,27$$

TABELAS

$$M_1 = 0,51135$$

$$T_1 = 486,21$$

Tabela 1.1 – Tabela para escoamento Isoentrópico. $k = 1,4$

M	$\frac{T_0}{T}$	$\frac{P_0}{P}$	$\frac{\rho_0}{\rho}$
0,00	1,000000000	1,000000000	1,000000000
0,02	1,000080000	1,000280028	1,000200012
0,04	1,000320000	1,001120448	1,000800192
0,06	1,000720000	1,002522269	1,001800972
0,08	1,001280000	1,004487173	1,003203073
0,10	1,002000000	1,007017518	1,005007502
0,12	1,002880000	1,010116340	1,007215559
0,14	1,003920000	1,013787360	1,009828831
0,16	1,005120000	1,018034982	1,012849194
0,18	1,006480000	1,022864304	1,016278817
0,20	1,008000000	1,028281121	1,020120160
0,22	1,009680000	1,034291935	1,024375975
0,24	1,011520000	1,040903957	1,029049309
0,26	1,013520000	1,048125123	1,034143503
0,28	1,015680000	1,055964098	1,039662194
0,30	1,018000000	1,064430286	1,045609318
0,32	1,020480000	1,073533846	1,051989110
0,34	1,023120000	1,083285700	1,058806103
0,36	1,025920000	1,093697545	1,066065136
0,38	1,028880000	1,104781869	1,073771352
0,40	1,032000000	1,116551966	1,081930199
0,42	1,035280000	1,129021948	1,090547435
0,44	1,038720000	1,142206766	1,099629126
0,46	1,042320000	1,156122222	1,109181654
0,48	1,046080000	1,170784991	1,119211715
0,50	1,050000000	1,186212638	1,129726322
0,52	1,054080000	1,202423640	1,140732810
0,54	1,058320000	1,219437404	1,152238835
0,56	1,062720000	1,237274290	1,164252381
0,58	1,067280000	1,255955636	1,176781759
0,60	1,072000000	1,275503776	1,189835612

$$\begin{array}{l} M \\ 0,56 \\ 0,51135 \\ 0,58 \end{array} \quad \begin{array}{l} P_0/P \\ 1,06212 \\ T_0/T \\ 1,06728 \end{array}$$

$$\frac{T_0/T - 1,06212}{1,06728 - 1,06212} = \frac{0,51135 - 0,56}{0,58 - 0,56}$$

$$\frac{T_0}{T_1} = 1,0666758$$

$$T_{02} = 518,69 \text{ K} \quad (518,688)$$

Choque Normal $M = 2$

$$M_2 = 0,57735026 \text{ (0,57735)}$$

$$\frac{P_2}{P_1} = 4,5 \text{ (4,5)}$$

$$\frac{T_2}{T_1} = 1,6875 \text{ (1,6875)}$$

$$\leadsto T_2 = 486,27$$

$$\frac{P_{02}}{P_{01}} = 0,72087386$$

$$T_2 = 486,27$$

$$\frac{P_{02}}{P_{01}} = 0,72087386$$

$T_{02} = ?$ Escoamento isentrópico

$$M_2 = 0,57735026$$

$$\frac{T_2}{T_{02}} = 0,9315$$

$$\approx T_{02} = \frac{486,27}{0,9315}$$

$$T_{02} = 518,688 \text{ (518,688)}$$

$$T_2 = 486,27$$

$$\frac{p_{02}}{p_{01}} = 0,72087386$$

p_{01} ? Escoamento isentrópico

$$M_1 = 2 \leadsto \frac{p}{p_0} = 0,12780452$$

$$p_{01} = \frac{101,325}{0,12780452} \leadsto p_{01} = 792,8123 \text{ kPa}$$

$$p_{02} = 571,5316 \text{ kPa } (571,5175)$$

Formulário e Tabelas disponíveis em

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The logo consists of a large yellow circle centered on a red background. Inside the circle, the text "CIÊNCIAS TÉRMICAS" is written in a bold, white, sans-serif font, with "CIÊNCIAS" on the top line and "TÉRMICAS" on the bottom line. Below this, the text ".com" is written in a smaller, white, sans-serif font.

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